> *"The time has come," the Walrus said,*
> *"To talk of many things:*
> *Of shoes--and ships--and sealing-wax--*
> *Of cabbages--and kings--*
> *And why the sea is boiling hot--*
> *And whether pigs have wings."*

And Froggy. Since publishing my solution[1] to Lewis Carroll's Froggy problem[2], a few questions have been niggling away at the back of my mind and, despite my insistence that they stay there, they have preferred to leave that shadowy refuge and seek fame and fortune in the warm light of day.

The first of these somewhat disobedient entities is the question as to why Carroll decided to start some of the Premisses in the Froggy problem with the word "When" and others with the word "Whenever". That the two words are different is not in dispute, but whether the *meanings* are different, in the context of the Froggy problem, is a more debatable question. Are they twins (though clearly not *identical* twins) or are they merely brothers? Premise **6** tells us that "When it is very hot, the thermometer is high" whereas Premiss **4** informs us sadly that "Whenever I go out on the roof to enjoy a quiet cigar, I'm sure to discover that my purse is empty". Why does this "ever" choose to add itself to the proud "When" which boasts a distinguished capital at its head? Does Carroll intend it simply to avoid the monotony of continuous repetition or is there a deeper purpose hidden within? In "The Problem of the School-Boys" [3], Each of the 12 Premisses starts with "Whenever" and so an aversion to repetition does not seem to be a Carroll priority and yet *"When it is very hot, the thermometer is high"* seems to exclude the possibility of the thermometer being not-high on any occasion when it is hot, just as adequately as *"Whenever I go out on the roof to enjoy a quiet cigar, I'm sure to discover that my purse is empty"* excludes the possibility of the purse containing anything at all on any occasion when I go out on the roof to enjoy a quiet cigar. So I have decided to relegate this particular niggling question to the category of "irrelevant, not wanted here" and will assume that "When" and "Whenever" are indeed twins having exactly the same meaning but with different outward appearance.

The second niggling thought is the strange and unexpected appearance, in the dictionary that is believed to belong to Froggy[4], of a Greek letter (ε when all the other symbols for attributes are from the Roman alphabet. Carroll tells us that the Universe for this problem is "Cosmophases" and that the ε is to represent the attribute "this". I suspect that we have here a glimpse into his method to treat with the Dated Copula[5] which either he never wrote or has been lost. It leaves us with the question of how to represent, in Carroll's symbolic notation, the temporal clauses that appear in Premisses **3**, **9** and **11**. It seems clear to me that, based on Carroll's comments in the "Raw Meat" problem[6] and his instructions in dealing with "a really long and hard Problem" [7], the correct representation of Premiss **3** in Froggy would be $\varepsilon_1(ak')'_0$ which can be written as $\varepsilon_1a'_0 \dagger \varepsilon_1k_0$ (so that **a** must be reckoned as appearing in *negative* form and **k** in positive form in this Premiss).

The third niggling thought is the full meaning of Premiss **3**, *"Now that Froggy's hair is out of curl, he has put away his gorgeous waistcoat"*. We are obviously entitled to conclude that "Now" (which is to say, in *this* cosmophase) it is both true that Froggy's hair is out of curl (attribute **a**) and also that Froggy is not wearing a waistcoat that is gorgeous beyond words (attribute **k'**). But are we also entitled to conclude that he has put away his gorgeous waistcoat precisely *because* his hair is out of curl and that whenever his hair is out of curl, he puts away his gorgeous waistcoat (a_1k_0)? I see no other reason for the words "Now that" at the beginning of the Premiss; *"Froggy's hair is out of curl and he has put away his gorgeous waistcoat"* would have expressed $\varepsilon_1(ak')'_0$ perfectly well but the addition of "Now that" seems to me to imply a causal relationship and so makes Premiss **3** in symbolic form $\varepsilon_1a'_0\dagger\varepsilon_1k_0\dagger a_1k_0$.

The same consideration applies to Premiss **11**, *"Now that it looks like rain, and Froggy is grinning like a hyena, I can do without my cigar"*. In *this* cosmophase (Now) it looks like rain, Froggy is grinning like a hyena and I am not taking a quiet cigar (**scn'**), but can we also conclude that "Whenever it looks like rain and Froggy is grinning like a hyena then I do not take a quiet cigar"? And of course we are here making two further assumptions: (1) that *"it looks like rain"* is the same as Carroll's dictionary item **s**, *"It is going to rain",* and (2) that *" I can do without my cigar"* is the same as the negative of Carroll's dictionary item **n**, *"I take a quiet cigar"*. I think that these two further assumptions must be accepted if we accept that the dictionary given is indeed the dictionary of the version of Froggy's problem that we have[4]. For otherwise there would be no way of writing this premise in symbolic form using the dictionary given. So, accepting that "Now that" implies a causal relationship would lead us to write Premiss **11** as $\varepsilon_1(scn')'_0\dagger sc_1n_0$ which Carroll tells us can be written as $\varepsilon_1s'_0\dagger\varepsilon_1c'_0\dagger\varepsilon_1n_0\dagger sc_1n_0$.

The fourth and most serious of the niggling thoughts concerns, in true English style, the weather. In the given dictionary we have only two attributes regarding the state of the weather: **s**, "it is going to rain" and **t**, "it is very hot". However, in the Premisses the following comments about the weather appear:

"the day is fine" in **1** and **7**

"it is very hot" in **6**

"it looks like rain" in **11**

"it's chilly" in **16**

"it is going to rain" in **18**

"the day is cool" in **20**

Of these, the attributes appearing in Premisses **6** and **18** are exactly those appearing in the dictionary (**t** and **s**) and therefore we can acknowledge their good behaviour and not

trouble them further. But what are we to make of the others? I think we must interpret "it looks like rain" as being the same as "it is going to rain" since otherwise we would have no way of dealing with this feature using Carroll's dictionary (and in any case the technology of weather forecasting at the time Carroll wrote this was unable to differentiate between the two - and probably still is!). But "it's chilly" in Premiss **16** is more troublesome. Premiss **11** tells us that I am not now (in *this* Cosmophase) "taking a quiet cigar" and also that "it looks like rain". We are also informed by Premiss **9** that "my railway shares are going up". So from **16** we must conclude that it isn't chilly now (in *this* Cosmophase) or otherwise we would have a contradiction. In symbols, if we use **X** to denote "it's chilly", we have (all in *this* Cosmphase) **16: w&X&s=>n** and from **11** we know **n'** and so we conclude (**w&X&s**)' which by DeMorgan´s law is **w'ᵥX'ᵥs'**. However, we already know **w** from **9** and **s** from **11** and so we are left with **X'**, "it isn't chilly now" (in this Cosmophase). If we interpret this as being the same as "it is very hot" (**t** in the dictionary) we must conclude from **6** that in *this* Cosmophase "the thermometer is high" and hence from **12** that "you need not trouble yourself to take an umbrella", which is, to say the least, oddly perverse given that we know from Premiss **11** that "it looks like rain". So it seems clear that our **X** ("it's chilly") is not the negation of **t** "it is very hot". The only sensible alternative, if we are not to introduce a new attribute into the dictionary, seems to be that "it's chilly" is the negation of "the thermometer is high" and our **X** is in fact **A'**. It is possible that this is the "beautiful 'trap' " to which Carroll referred in his diary entry for September 5, 1896.[8]

What, then, are we to make of "the day is fine" and "the day is cool"? Clearly "the day is cool" is not to be considered the negation of "the thermometer is high" since in that case where it states in Premiss **20** "When the day is cool and the thermometer low" it would be repeating the same attribute twice. So the only alternative for this troublesome fellow is to assume he is the contrary to "it is very hot" and should be represented by **t'**. Which leaves us with "the day is fine" and I think the only space left for *him* is as the negation of "it is going to rain" and therefore in symbols **s'**. (See "Note added 31 July 2020" at the end of this document)

The fifth, and final, niggling thought concerns Premiss 10. What are we to make of "things are apt to get rather warm"? I recall when I was a child it was fairly common to hear "the temperature rose" when one spoke of a discussion becoming heated, so I am inclined to think that we should interpret " things are apt to get rather warm" as "the thermometer is high" rather than as "it is very hot". However, as we shall see, the choice of which interpretation to use does not affect the outcome.

Having dealt with these niggling questions we can now put the Froggy problem into symbolic form, using the dictionary in the way we have decided above. The Premisses in symbolic form and the corresponding Register are as follows:

PREMISSES		REGISTER				
1	$s'_1r'_0$					
2	$m'h_1d_0$	a	b	c	d	e
3	$\varepsilon_1a'_0\dagger\varepsilon_1k_0\dagger a_1k_0$	3		5, 7, 11	2, 17	19
4	$ln_1v'_0$					
5	zm_1c_0	3, 15	18	11, 14, 20		17
6	$t_1A'_0$					
7	$s'n'c_1r_0$	h	k	l	m	n
8	$zv_1m'_0$	2	3, 10, 13	4	5, 10	4, 11, 18, 20
9	$\varepsilon_1w'_0$					
10	$vkm_1A'_0$	13, 15	17	15	2, 8	
11	$\varepsilon_1s'_0\dagger\varepsilon_1c'_0\dagger\varepsilon_1n_0\dagger sc_1n_0$					7, 13,16
12	A_1B_0	r	s	t	v	w
13	$kh'_1n'_0$	7, 14	11, 16, 18	6	8, 10	16, 19
14	$r_1c'_0$					
15	$v'a'h'_1l'_0$	1, 20	1, 7, 11	20	4, 15	9
16	$wA's_1n'_0$					
17	$d_1e'_0\dagger d_1k'_0$	z	A	B	ε	
18	$snb'_1B'_0$	5, 8	12	12	3, 9, 11	
19	$we_1z'_0$					
20	$t'A'r'c'_1n_0$					

Splitting premisses 3,11 and 17 into their constituent parts we have 26 premisses and 16 eliminands. We have now only 3 retinends, ε, **b** and **d** and the solution *ought* to be **εb'd$_0$.** And with 26 premisses and 16 eliminands there might be 9 superfluous premisses.

So if we can prove this conclusion we will have shown that Froggy intends to go a-wooing if he has his mother's permission (in *this* Cosmophase at any rate). Note that using **t'** instead of **A'** in Premiss 10 does not change the retinends and therefore the solution is not affected by our interpretation of "things are apt to get rather warm" in Premiss **10**.

However, using the methods I have used[9] to solve most of the remaining problems that appear in "Problems and Exercises 1-83" [10] I can now make similar comments to those made by Carroll in his letter to his sister Louisa Dodgson, dated 18 November 1896 [11] regarding the "M.P." problem. I can now say the following:

My concluding remark will, I expect, surprise you. **εb'd$_0$** is not the Complete Solution: and there are 9 superfluous premisses! The Complete Solution is in fact **εd$_0$** . This is a larger Conclusion than **εb'd$_0$**, as it tells us, not only that **εb'd$_0$** is a Nullity, but also that

$\mathbf{\epsilon bd_0}$ is so. And with 26 premisses and 16 eliminands there might be 9 superfluous premisses. The method I have used[9] confirms this to be the case.

So the Complete (though allowing of disobedience) Solution is "Froggy does not now have his mother's permission to go a-wooing". The following 9 premisses are superfluous: the $\mathbf{\epsilon_1 k_0}$ part of 3; 6; 10; $\mathbf{\epsilon_1 s'_0 \dagger \epsilon_1 n_0}$ in 11; 12; 16; 18; 20.

This is all the more satisfactory since, in his letter to his sister, Carroll states "you've fallen into the trap that I am still hoping Professor Cook Wilson will fall into" and in his diary entry for September 5, 1896 he says "Finished, after about two days' work, my Sorites-Problem abt. "Froggy" which contains a beautiful 'trap'." [8]

It appears that the two "traps" are similar.

Method[9] applied to other problems

I have applied the same methods I used for Froggy to two other problems - The Pork-Chop Problem[11] and the Members of Parliament Problem[12] .

The method could aptly be described as the "brute-force" method in that it is based on considering all possible cosmophases and applying to each one in turn to all of the premisses to see which cosmophases are not consistent with one or more premisses. This procedure identifies all of the nullities involving the retinends and also identifies the redundant premisses.

In the case of the Pork-Chop Problem (using the 18-premiss Version II from the first edition of Part I of *Symbolic Logic*, given on page 333 of Bartley), the result is that premiss 3 is redundant. This may explain the reference to a 17 premiss version that Carroll mentions in his letter of October 24, 1896 to Cook Wilson. Removing the attribute "young" (attribute *s* in the dictionary attached to the Version II problem) also removes two further premisses and produces the 15 premiss version which appears in the Appendix to the fourth edition of Part I of *Symbolic Logic*. It is worth noting that only premiss 3 is redundant in the original problem. The removal of the other two premisses to produce the 15 premiss version results from noting that the omission of the attribute "young" does not change the solution; but this is of course a different problem from the original 18-premiss version.

In the case of the Members of Parliament Problem the method shows that premiss 12 is redundant and also that there are two nullities, *kvw* and *kv'w* , which is exactly what Carroll says in his letter of November 18, 1896 to Louisa Dodgson although he doesn't explain how he arrived at that conclusion; he merely states "use *kw* as the Root, & ignore 2,12,14,17". His justification for so doing is that "it's worth while, as v' only appears in 2 premisses, to see if we ca'n't ignore it". The method I have used shows from the outset that *kvw* and *kv'w* are both nullities and therefore that kw_0 is the correct root to use.

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8 March 2013.

Notes:

1. http://www.connectedglobe.com/froggy/index.html
2. *APPENDIX, ADDRESSED TO TEACHERS. § 10. Some account of Parts II, III.Problem 3.* in "Symbolic Logic" by Lewis Carroll
3. *Book XIII, Chapter III The Problem of the School-Boys* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III. ISBN: 0-85527-984-2
4. *Book IV, Problem 30* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III. (Bartley states "Only this fragment of Example 30 remains. It goes with Chapter V (Froggy) in Book XIII". This leaves open the possibility that the dictionary we have may not be the dictionary for the version of Froggy that we have. It may be that revisions were made to the Froggy problem which have not survived.
5. *Book XXI, Chapter IV, Solution of Classical Puzzles. [§6] Raw Meat* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.
6. *Book XXI, Chapter II, Classical Puzzles. [§6] Raw Meat* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.
7. *Book XII, Chapter III, Sorites-Problems. page 305* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.
8. Lewis Carroll's Diaries, The Private Journals of Charles Lutwidge Dodgson Volume 9 Ed. Edward Wakeling.
9. The method that was used to find the Complete Conclusion of the Froggy problem, its implementation and its application to the remaining problems of *Book XIV, Some Further Problems to be Solved by the Methods of Part II* will be described fully in a forthcoming book by Graham Hawker.
10. *Book XIV, Some Further Problems to be Solved by the Methods of Part II.* page 385 in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.
11. *Book XIII, Chapter IV, The Pork-Chop Problem. page 331* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.
12. *Book XIII, Chapter VI, Members of Parliament Problem. page 343* in "Lewis Carroll Symbolic Logic" Ed. William Warren Bartley, III.

Note added 31 July 2020

The movement constraints of the Covid19 pandemic have given me the opportunity to review thie work I did on the Froggy problem. In particular I have been thinking more about the interpretation of some of the phrases used by Carroll.

Proposition 1. states:

When the day is fine, I tell Froggy "You're quite the dandy, old chap!";

and proposition 7 states

When the day is fine, and I'm not in the humor for a cigar, and Froggy is grinning like a hyena, I never venture to hint that he's quite the dandy;

These appear to be contradictory but of course on closer examination they simply imply that the situation

I'm not in the humor for a cigar, and Froggy is grinning like a hyena

is never true when the day is fine.

However, proposition 11 states that:

Now that it looks like rain, and Froggy is grinning like a hyena, I can do without my cigar

so clearly we cannot assume that "the day is fine" is the negation of "it looks like rain" wthout creating another contradiction. But the phrase "it looks like rain" does not appear in the dictionary so what exactly did Carroll mean?

Examining carefully the phraseology used in the dictionary we note:

B=you had better take an umbrella

This is the phrase one would use to suggest "just in case it rains" and in the case of "it is going to rain" the phrase would be "you will need an umbrella".

If we accept this interpretation we should probably assume that "it looks like rain" is synonymous with "you had better take an umbrella" and should therefore be represented by "B" from the dictionary.

And this helps resolve the other phrase which appears in the problem but not in the dictionary:

the day is fine

This would not be what one would say if it looked like rain and so it would be sensible to consider it as the negation of "it looks like rain" and so we should consider "the day is fine" to be the negation of B